Bridging the Gap Between Position-Based and Content-Based Self-Attention for Neural Machine Translation

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Abstract

Position-based token-mixing approaches, such as FNet and MLPMixer, have shown to be exciting attention alternatives for computer vision and natural language understanding. The motivation is usually to remove redundant operations for higher efficiency on consumer GPUs while maintaining Transformer quality. On the hardware side, research on memristive crossbar arrays shows the possibility of efficiency gains up to two orders of magnitude by performing in-memory computation with weights stored on device. While it is impossible to store dynamic attention weights based on token-token interactions on device, position-based weights represent a concrete alternative if they only lead to minimal degradation. In this paper, we propose position-based attention as a variant of multi-head attention where the attention weights are computed from position representations. A naive replacement of token vectors with position vectors in self-attention results in a significant loss in translation quality, which can be recovered by using relative position representations and a gating mechanism. We show analytically that this gating mechanism introduces some form of word dependency and validate its effectiveness experimentally under various conditions. The resulting network, rPosNet, outperforms previous position-based approaches and matches the quality of the Transformer with relative position embedding while requiring 20% less attention parameters after training.[1]

1 Introduction

The Transformer (Vaswani et al., 2017) revolutionized the field of neural machine translation before its wide adoption in numerous other tasks (Dong et al., 2018; Devlin et al., 2019; Chen et al., 2021; Dosovitskiy et al., 2021). Using self-attention (Vaswani et al., 2017), the Transformer computes high-level representations for each token as a weighted sum of the entire sequence, where the weights depend on the pairwise content interactions. However, recent work argues that results similar to the Transformer can also be achieved by modeling self-attention weights based on positional instead of content information (Wu et al., 2019; You et al., 2020; Tolstikhin et al., 2021; Liu et al., 2021; Lee-Thorp et al., 2022). Often, these position-based methods are used with some form of gating mechanism that precedes or wraps the token-mixing operation (Wu et al., 2019; Liu et al., 2021; Kim et al., 2023).

Position-based self-attention alternatives often speed up the computation on commercial computing devices like GPU, but they can become more attractive from the perspective of using memristive crossbar arrays (Chua, 1971; Strukov et al., 2008). Recent advances in analog in-memory computation with memristive crossbar arrays have shown impressive efficiency improvements in the inference of deep learning models (Hu et al., 2018; Wang et al., 2019; Kataeva et al., 2019; Yao et al., 2020; Xue et al., 2021), up to 110 times better energy efficiency and 30 times better performance density compared to a Tesla V100 GPU (Yao et al., 2020). However, such efficiency is obtained by storing weights of matrix-vector multiplications in the device rather than calculating them on the fly, which excludes the possibility of using attention to compute the weight matrix.

With the goal of finding self-attention alternatives for machine translation that can be more easily used with memristive crossbar arrays, we compare existing position-based approaches and observe a significant quality loss when they use no form of gating. Additionally, by scoring with a diverse set of metrics, we show that, even with gating, no existing approach can consistently match Transformer results. While the role of gating to guide the information flow of neural networks is known (Srivastava et al., 2015; Dauphin et al., 2017), its

[1] Code available at https://github.com/apptek/posnet-position_based_attention

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importance for the performance of position-based approaches has yet to be explored.

In this paper, we propose aPosNet and rPosNet, two position-based networks that leverage gating and compute self-attention weights based on the interactions of **a**bsolute and **r**elative position representations. Both differ slightly from the Transformer baseline with relative position embeddings (Shaw et al., 2018), which enables us to deliver insights into gating and its dependency on position information. In summary, we provide the following contributions:

- Analytically, we derive that wrapping the weighted sum of tokens with a gating mechanism introduces latent content-dependent token-mixing weights (Section 3).
- We provide an inference-time matrix precomputation for positional attention that can be easily stored in device (Section 4).
- rPosNet outperforms existing position-based methods and performs on par with the Transformer with relative position embeddings while saving 20% of the self-attention parameters (Section 6).
- We show that increasing the expressiveness of token-mixing weights reduces the usefulness of gating, coherently with the idea that it enables content-based interactions (Section 7.1).
- We observe experimentally that rPosNet is less effective when used in cross-attention. Our gating reformulation suggests one probable reason, but we leave detailed investigations for future work (Section 7.2).

2 Background

Neural machine translation is typically modeled with an encoder-decoder sequence-to-sequence (Sutskever et al., 2014) Transformer, which mainly consists of multi-head attention and feed-forward sub-layers. In the following, we introduce our notation, position-based token-mixing alternatives and the gating mechanism commonly used in modern architectures.

2.1 Multi-head attention

Given a source sequence representation $\mathbf{x} \in \mathbb{R}^{M \times D}$ and target sequence representation $\mathbf{y} \in \mathbb{R}^{N \times D}$, the multi-head attention mechanism (Vaswani et al., 2017) mixes the elements in $\mathbf{x}$ for every element in $\mathbf{y}$. If $\mathbf{y}$ and $\mathbf{x}$ refer to the same sequence, it is called self-attention. The multi-head concept derives from performing the following operations on H parallel splits of the feature dimension D. In this work, we drop the head indices for simplicity of notation. To calculate the unnormalized mixing weight, referred to as attention energy, of y_n and x_m, those are projected into query and key and combined using the dot product:

$$\hat{\alpha}_{nm} := \frac{(W^Q y_n)(W^K x_m)^\top}{\sqrt{D}}. \tag{1}$$

Since $\hat{\alpha}_{nm}$ is computed from token contents, we say that attention captures token-token interactions. The attention weight is then calculated by the softmax normalization of the attention energy:

$$\alpha_{nm} := \frac{\exp \hat{\alpha}_{nm}}{\sum_{m'} \exp \hat{\alpha}_{nm'}}, \tag{2}$$

and used as the token-mixing weight in the weighted sum over projected input tokens $\mathbf{x}$, denoted value vectors:

$$c_n := \sum_m \alpha_{nm} \cdot (W^V x_m). \tag{3}$$

We will refer to the result c_n as context vector. Finally, the context vectors of each head are concatenated and mixed with a linear projection, called output projection.

2.2 Position-based token mixing

We briefly overview how existing position-based token-mixing approaches propose to modify the attention weights and provide the corresponding Equations in Appendix A for comparison.

FNet Proposed for language understanding, FNet (Lee-Thorp et al., 2022) applies a 2D Fourier transform over the spatial and feature dimension of $\mathbf{x}$. However, this formulation performed poorly in preliminary experiments, which is why our FNet implementation, denoted FourierNet, applies a 1D Fourier transform along the spatial dimension and employs value and output projections. We will show in our results that, despite its claimed good quality for natural language understanding, the translation quality achieved by FourierNet is significantly lower than Transformer.

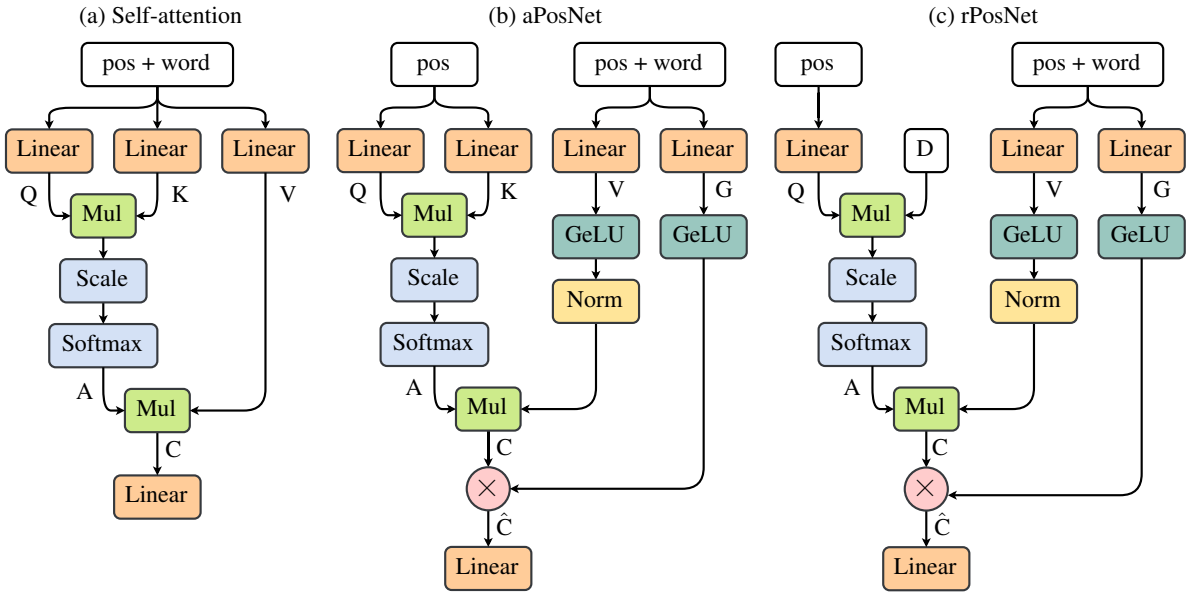

Figure 1: Flowchart representation of (a) self-attention, (b) gated absolute position-based attention (aPosNet), and (c) gated relative position-based attention (rPosNet). While self-attention provides word and position information to queries (Q) and keys (K), we omit word information to calculate the attention weights of PosNet. In rPosNet, we model relative positions using relative position representations (D). In addition, we employ the gating mechanism presented in Section 2.3, which applies GeLU activation and layer normalization on the values (V) and elementwise multiplies the context vector (C) with the GeLU activated gate (G) resulting in the gated context vector (Ĉ).

GaussianNet Proposed for machine translation, You et al. (2020) hardcode self-attention weights as a Gaussian distribution. They report similar performance to the Transformer when GaussianNet is applied for self-attention but a significant degradation if extended to cross-attention.

LinearNet Tolstikhin et al. (2021) propose mixing tokens with a learnable spatial projection, effectively representing α. It has been proposed, together with other architecture changes, for image classification and natural language understanding with minor degradations to the Transformer.

LightConv For machine translation and other tasks, Wu et al. (2019) introduce a lightweight form of depthwise convolution, which shares the kernel weights W across the feature dimension of a head and the outputs while additionally softmax normalizing them.

gLinearNet Liu et al. (2021) combine the spatial projection of LinearNet with the gating mechanism of Section 2.3. They propose their architecture for image classification and masked language modeling and report significant improvements over Tolstikhin et al. (2021).

2.3 Gating mechanisms

Various formulations of gating mechanisms have been proposed to control the information flow in neural networks (Hochreiter and Schmidhuber, 1997; Cho et al., 2014; Srivastava et al., 2015; van den Oord et al., 2016; Dauphin et al., 2017). They all have in common an elementwise multiplication between two vectors where one, the gate, is bounded in the $[0, 1]$ interval. The gating mechanism we consider here has been proven effective with position-based token-mixing approaches (Liu et al., 2021; Kim et al., 2023) and differs from other gating mechanisms in that the gate is GeLU activated (Hendrycks and Gimpel, 2016) and thus only lower bounded. This gating mechanism modifies the weighted sum of Equation 3 by applying layer normalization (Ba et al., 2016) on the value vector $v_m = W^V x_m$ and elementwise multiplying the context vector with the gate $g_n = \sigma_g(W^G y_n)$:

$$\hat{c}_n := \Big[\sum_m \alpha_{nm} \cdot \mathrm{Norm}\big(\sigma_g(v_m)\big)\Big] \odot g_n, \quad (4)$$

where σ_g refers to the GeLU function and $\hat{c}_n$ to the gated context vector. In general, gating can be applied with any formulation of α. However, we will show experimentally in Section 7.1 that its benefits

strongly depend on the information incorporated within α.

3 Reformulating the Gating Mechanism

To better understand the implications of gating, we reformulate Equation 4. We omit layer normalization for simplicity and will show in the Appendix B that the general reformulation is unaffected if we apply layer normalization on v_m. Additionally, we leverage the GeLU approximation $\sigma_g(v_m) \approx v_m \sigma_s(1.702 v_m)$, where σ_s refers to the Sigmoid function, and rewrite Equation 4 as

$$\hat{c}_n \approx \sum_m \alpha_{nm} \beta_{nm} \odot v_m. \tag{5}$$

Equation 5 shows that gating the context vector introduces the latent weights $\beta_{nm} \in \mathbb{R}^D$:

$$\beta_{nm} = g_n \odot \sigma_s(1.702 v_m), \tag{6}$$

which consists of the two independent factors $\beta'_n = g_n$ and $\beta'_m = \sigma_s(1.702 v_m)$. While the multiplication of β'_n and β'_m, in general, allows for token-token interactions, the independence of these factors poses a limitation: for a given query token y_n, the ratio between the weights assigned to x_m and to $x_{m'}$ is independent of y_n:

$$\frac{\beta_{nm}}{\beta_{nm'}} = \frac{\beta'_m}{\beta'_{m'}}. \tag{7}$$

In other words, the ratios of token-mixing weights for a query y_n as computed by β are predetermined by the ratios across $\beta'_{1\ldots M}$. While we show in Section 6 that this limitation is not problematic for self-attention, it may be part of the reason gating and relative position-based attention are not effective in cross-attention (see Section 7.2).

4 Position-based Attention

In this Section, we propose position-based attention, which determines the token-mixing weight connecting tokens x_m and y_n solely based on the position-position interactions between n and m. We pair position-based attention with the gating mechanism of Section 2.3.

4.1 Absolute position-based attention

In absolute position-based attention we compute the attention energy as the dot product between the two projected position embeddings $\tilde{n}$ and $\tilde{m}$:

$$\hat{\alpha}_{nm} := \frac{(W^Q \tilde{n})(W^K \tilde{m})^\top}{\sqrt{D_h}}. \tag{8}$$

While $\tilde{n}$ and $\tilde{m}$ are shared across all layers, W^Q and W^K are layer-specific. We refer to the combination of Equation 8 and the gating mechanism of Section 2.3 with aPosNet.

Pre-computing the attention energies The query and key inputs $\tilde{n}$ and $\tilde{m}$ are independent of the word representations y_n and x_m and are constant after training. Since the attention energy values $\hat{\alpha}_{nm}$ only depend on $\tilde{n}$ and $\tilde{m}$, we can pre-compute $\hat{\alpha}$ and obtain a matrix of the form $(H \times N \times M)$ that can be used during inference. In the following theoretical complexity discussions we set $N = M$ for simplicity of notation.

Theoretical complexity Apart from the gating overhead, aPosNet has similar theoretical complexity as attention. However, by pre-computing $\hat{\alpha}$, we can skip the dot-product and key query projections, reducing[2] the number of parameters from $5D^2$ to $HN^2 + 3D^2$ and the number of operations from $2N^2D + 5ND^2$ to $N^2D + 3ND^2$. We compare theoretical complexities in Appendix C.

Relation to gLinearNet With the pre-computed attention energy matrix, aPosNet becomes similar to gLinearNet except that α of gLinearNet is not normalized and has been trained directly.

4.2 Relative position-based attention

To model position interactions with relative position-based attention, we borrow the relative position representations $\tilde{d}_{nm}$ from Shaw et al. (2018), which we use in the dot product with the projected position embedding $\tilde{n}$:

$$\hat{\alpha}_{nm} := \frac{(W^Q \tilde{n})(\tilde{d}_{nm})^\top}{\sqrt{D_h}}. \tag{9}$$

Similarly to Shaw et al. (2018), the distance embedding $\tilde{d}$ is clipped to a maximum unidirectional context size K:

$$\tilde{d}_{nm} := \text{Embedding}_{\text{rel}}\Big(\text{clip}\big(\lfloor \gamma n \rfloor - m, K\big)\Big). \tag{10}$$

However, in contrast to Shaw et al. (2018), we extend relative position-based self-attention to be compatible with cross-attention by multiplying n with the length ratio $\gamma := \frac{M}{N}$ which we determine similar to You et al. (2020) by measuring the average length ratio on the training set. We refer to the

[2] Typically in sentence-level machine translation we have $N \ll D$.

Table 1: Dataset statistics.

Dataset	Vocab. Size		Train Pairs	Test Pairs	Valid Pairs
	Src	Tgt			
DE→EN	10k		160k	6750	7283
EN→DE	44k		4M	3003	40k
EN→FR	46k		36M	3003	27k
EN→ZH	32k	45k	17M	2001	13k

combination of Equation 9 and the gating mechanism of Section 2.3 with rPosNet. In Figure 1, we illustrate the operations performed by aPosNet and rPosNet in comparison to multi-head self-attention.

Pre-computing the attention energies Similar to aPosNet, we can pre-compute $\hat{\alpha}$ of rPosNet after training, which summarizes the interactions between query and relative position representations into a matrix of shape $(H \times \hat{K} \times N)$, where $\hat{K} = 2K + 1$. While the attention energy matrix of aPosNet grows quadratically with the length of the sequence, rPosNet's matrix grows linearly due to the constant size $\hat{K}$ of the relative position representations.

Theoretical complexity Pre-computing $\hat{\alpha}$ after training reduces the number of parameters from $\hat{K}D + 4D^2$ to $H\hat{K}N + 3D^2$ and operations from $\hat{K}ND + N^2D + 4ND^2$ to $N^2D + 3ND^2$. Inserting the Base model configuration of Section 5 ($D = 2048$, $\hat{K} = 33$) and the maximum sentence length $N = 128$, this pre-computation of $\hat{\alpha}$ saves 23% of attention parameters.

Relation to LightConv After pre-computing $\hat{\alpha}$, rPosNet differs from LightConv in that rPosNet's weights have global context and depend also on the absolute query position, and as such are not shared across y_n. We provide an ablation study in Section 7.3 to understand the importance of these differences.

5 Experimental Setup

5.1 Datasets & evaluation

We perform our comparison on four datasets of varying sizes: IWSLT14 German-English (Federico et al., 2014), WMT14 English-{German, French} (Bojar et al., 2014), and WMT18 English-Chinese (Bojar et al., 2018). We split each dataset into train and validation pairs and evaluate DE→EN models on the test sets TED-{dev10,dev12, test10, tst11, tst12}, EN-{DE, FR} models on newstest14 and EN→ZH models on newstest17. An overview of the dataset statistics is shown in Table 1. We preprocess all datasets using Byte Pair Encoding (BPE) (Sennrich et al., 2016) and lowercase the text for the DE→EN direction.

We report BLEU (Papineni et al., 2002), BLEURT (Sellam et al., 2020), and COMET (Rei et al., 2020) for each evaluation. All scores are calculated on detokenized text. To calculate BLEU scores, we use sacreBLEU[3] and its internal tokenizations[4][5]. For BLEURT and COMET, we use the official implementations[6][7] and the models *BLEURT-20* and *wmt20-comet-da*, respectively. To summarize results, we will refer to the translation quality difference between two approaches as their relative difference averaged across all metrics and datasets.

5.2 Model architectures

Our Base and Big Transformer architectures follow the implementation of Vaswani et al. (2017), whereas, for the Small models, we halve the feed-forward dimension to 1024 and increase dropout to 0.3. We compare position-based token-mixing approaches by leveraging the respective formulations instead of encoder/decoder self-attention while leaving the rest of the Transformer architecture unchanged. We make an exception for FourierNet, which cannot be straightforwardly extended to the decoder because it has an explicit dependency on the sequence length. Instead, FourierNet uses multi-head attention within decoder self-attention.

In preliminary experiments, we found that aPosNet works best with sinusoidal positional embeddings (Vaswani et al., 2017) and rPosNet with learnable embeddings (Gehring et al., 2017). All other position-based token-mixing approaches use sinusoidal positional embeddings. Similar to Shaw et al. (2018), our implementation of rPosNet and LightConv use a unidirectional context window $K = 16$ for the Base and $K = 8$ for the Big model.

5.3 Training setup

Our training setup closely follows the configuration of Vaswani et al. (2017). Similarly, we use

[3] https://github.com/mjpost/sacrebleu

[4] SacreBLEU signature for EN, FR, DE: nrefs:1|case:mixed|eff:no|tok:13a|smooth:exp|version:2.0.0

[5] SacreBLEU signature for ZH: nrefs:1|case:mixed|eff:no|tok:zh|smooth:exp|version:2.0.0

[6] https://github.com/google-research/bleurt

[7] https://github.com/Unbabel/COMET

Table 2: Base model results on EN→DE, EN→FR, and EN→ZH. We calculate statistical significance (p-value $\leqslant 0.05$) using paired bootstrap resampling with respect to the Transformer (†) and to Shaw et al. (2018) (‡). Note that this scoring differs from Vaswani et al. (2017) in that they split German compound words, which usually increases the BLEU score, and from You et al. (2020) in that we use sacreBLEU's default tokenizer, not 'intl'. We ensured that our baseline system and reimplementation of You et al. (2020) match in BLEU when evaluating similarly.

Model	Params	EN→DE			EN→FR			EN→ZH		
	(EN→DE)	BLEU	BLEURT	COMET	BLEU	BLEURT	COMET	BLEU	BLEURT	COMET
Transformer	66.5M	26.3	71.1	47.6	37.8	69.0	61.1	33.8	64.3	42.5
Shaw et al. (2018)	66.7M	26.3	**71.4**	**48.6**	37.8	69.2	61.6	**34.0**	**64.6**	**43.5**
FourierNet	63.4M	22.8†‡	66.0†‡	31.8†‡	34.9†‡	64.2†‡	49.3†‡	31.5†‡	61.6†‡	34.9†‡
GaussianNet	60.2M	25.3†‡	68.1†‡	39.5†‡	36.7†‡	66.9†‡	55.7†‡	32.6†‡	62.6†‡	36.8†‡
LinearNet	61.8M	25.3†‡	69.8†‡	44.3†‡	37.0†‡	67.7†‡	58.2†‡	33.1†‡	63.3†‡	40.2†‡
LightConv	63.4M	26.0†‡	70.6†‡	46.7†‡	37.4†‡	68.6†‡	60.3†‡	33.0†‡	63.5†‡	41.1†‡
gLinearNet	65.0M	26.1	70.8‡	46.7‡	37.8	69.1	61.3	33.5‡	64.0‡	42.4‡
aPosNet	65.0M	25.9†‡	70.6†‡	46.1†‡	37.7	69.0	61.4	33.6‡	63.7‡	42.2‡
rPosNet	63.9M	**26.6**	**71.4**†	**48.6**	**37.9**	**69.4**†	**61.8**	33.8	64.2	43.1

Table 3: Big model results on EN→DE and Small model results on DE→EN.

Model	EN→DE				DE→EN			
	Params	BLEU	BLEURT	COMET	Params	BLEU	BLEURT	COMET
Transformer	221M	27.1	72.3	50.4	36.8M	35.0	69.3	37.6
Shaw et al. (2018)	221M	**27.3**	**72.7**†	**51.5**†	37.0M	**35.4**†	**69.7**†	**38.8**†
FourierNet	208M	24.0†‡	67.6†‡	36.5†‡	33.6M	32.5†‡	66.9†‡	28.2†‡
GaussianNet	196M	26.3†‡	69.4†‡	42.3†‡	30.4M	34.3†‡	68.4†‡	34.1†‡
LinearNet	199M	26.6†‡	71.3†‡	48.0†‡	32.0M	34.0†‡	68.3†‡	33.9†‡
LightConv	209M	26.8†‡	71.7†‡	49.1†‡	33.6M	34.4†‡	68.9†‡	35.5†‡
gLinearNet	212M	27.1	72.2‡	49.9‡	35.2M	34.5†‡	69.0†‡	36.3†‡
aPosNet	212M	26.8†‡	71.4†‡	47.7†‡	35.2M	34.2†‡	68.5†‡	34.7†‡
rPosNet	210M	**27.3**	72.2‡	50.4‡	34.1M	35.1‡	69.5‡	38.2‡

the Adam optimizer (Kingma and Ba, 2014) and a warmup learning rate schedule with 4000 steps. We group batches by sentence length and train the Small models for 30k steps, the Base models for 150k, and the Big models for 300k.

The final model is an average over the best checkpoint and its following if there are enough checkpoints to average, or else we take an average over the last checkpoints. We determine the best checkpoint by its perplexity on the validation set. For DE→EN, we consistently average 30 checkpoints with a checkpoint period of 300 steps; for the Base models, we average 7 checkpoints with 1000 steps each; for the Big models, 20 checkpoints with 600 steps each.

The Small models use an effective batch size of approximately 16000 target tokens while the Base and Big models accumulate approximately 27000 target tokens per step. The source and target sentence lengths are restricted to 128 tokens. We use beam search with a beam size of 12 for all models. All models in this work are implemented in PyTorch (Paszke et al., 2019). The Small models are trained on a single 2080 TI graphics card, the Base models on two, and the Big models on four.

6 Results

We compare translation quality of the Base model configurations in Table 2, and Small and Big model configurations in Table 3.

Gated position-based attention In all experiments, we observe rPosNet performing as well or slightly better than the Transformer with an average translation quality increase of 0.7% across

all test sets and metrics. It shows that the self-attention weights of rPosNet, consisting of content-dependent β and position-dependent α, achieve sufficient expressiveness for machine translation. aPosNet cannot match this expressiveness and underperforms the Transformer with an average relative degradation of 1.8%. In the Small setup on DE→EN, this reaches an absolute degradation of 2.9 points in COMET and 0.8 points in BLEURT. The significant difference between aPosNet and rPosNet highlights the importance of relative position information in α.

The results of gLinearNet and LightConv further emphasize the strong modeling capabilities of absolute (query) and relative position (key) interactions in rPosNet. In comparison, token-mixing weights in gLinearNet solely model absolute position interactions and in LightConv relative position interactions. Both cannot match rPosNet's translation quality, with gLinearNet on average lacking behind by 1.3% relative and LightConv by 2.4%. Note that in contrast to Wu et al. (2019), we do not match parameters between LightConv and the Transformer. Most prominent in the Base setting on EN→DE, rPosNet outperforms gLinearNet by 0.6 BLEURT and 1.9 COMET points. While aPosNet cannot match Transformer results, rPosNet consistently outperforms other position-based methods and is on par with Shaw et al. (2018) and the Transformer across most model sizes and data conditions.

Hard-coded token-mixing weights Our results show that hard coding encoder self-attention weights as the twiddle factors of the Fourier transform (FourierNet) leads to poor results for machine translation and, on average across all datasets and metrics, degrades translation quality relative to the Transformer by 13.2%. In GaussianNet, weights are manually designed to follow the normal distribution of Transformer self-attention patterns, which significantly reduces the degradation to 6.3%. However, the translation quality is still considerably worse than LinearNet's, the weakest model with trainable self-attention weights. The difference between LinearNet and GaussianNet is negligible in BLEU but made visible with BLEURT and COMET, which correlate better with human judgment (Kocmi et al., 2021). In particular, we confirmed by manually analyzing a sample of translations (see Appendix E) that the semantic metrics discriminate better between translation hypotheses when they all have little overlap with the references

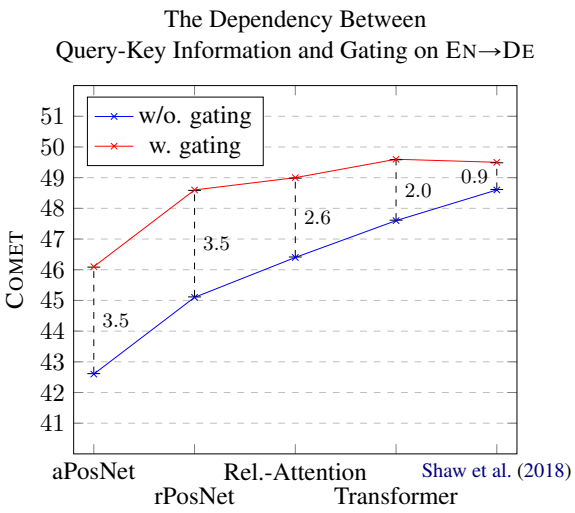

Figure 2: Approaches depicted on the x-axis differ in the provided information to queries and keys. On the y-axis we depict COMET, which is the most accurate metric according to Kocmi et al. (2021), and provide the full Table showing BLEU, BLEURT, and COMET in Appendix D. If position information is provided to queries and keys, gating has a significant positive impact on translation quality that diminishes with the usage of content information.

or changing a single word alters the meaning of the sentence. Thus, approaches with learnable token-mixing weights, such as rPosNet, are considerably better than hard-coded approaches.

7 Analysis

7.1 The impact of gating and query-key information

The gating mechanism is known to guide the learning of cross-token patterns (Tu et al., 2017; Dauphin et al., 2017). In Section 3, we mathematically showed that by gating the context vector, these patterns are captured within the latent token-mixing weights β. Since the products of β and α form the actual token-mixing weights, we analyze in this Section how content information in α impacts the usefulness of gating. For that, we compare the utilization of position versus content information in the query and key input of self-attention, with and without gating. The results are visualized in Figure 2, where we depict COMET scores on the y-axis and the query and key input on the x-axis.

The formulation of position-based attention without gating primarily[8] differs from the Transformer in the provided information within queries and

[8]The position embeddings may also differ between approaches.

keys. Relative attention uses the relative position representations of Shaw et al. (2018)'s approach but without the query-key dot product of multi-head attention. Thus, relative attention differs from rPosNet in that content information is provided to the queries and is equal to dynamic convolutions (Wu et al., 2019) with global context (Chang et al., 2021). In total, the x-axis of Figure 2 depicts position-position interactions for aPosNet and rPosNet, token-position interactions for relative attention, token-token interactions for the Transformer, and token-token + token-position interactions for Shaw et al. (2018). We sort these approaches on the x-axis in order of their attention weight expressiveness.

Figure 2 shows that the gating mechanism of the position-based attention approaches aPosNet and rPosNet increases COMET by 3.5 points. On the other hand, content-based approaches leverage gating with a lower absolute COMET increase of 2.6 points for relative attention, 2 points for the Transformer and only 0.9 points for Shaw et al. (2018). Thus, gating is less helpful if α can capture content-dependent patterns, and increasing the expressiveness of those patterns diminishes the usefulness of gating. Since gating introduces an additional projection matrix of size D^2 per self-attention layer, content-based mixing approaches may just leverage the additional parameters, but we leave further investigation for future work. In contrast, approaches that do not incorporate content information within the attention weights can benefit from token-token interactions captured in β. Additionally, the comparable performance of rPosNet and relative attention with gating suggests that gating makes the content information within relative attention redundant for translation quality.

7.2 Comparing the usage of rPosNet across attention layers

While the aforementioned experiments concentrated on self-attention, we also consider cross-attention in this Section and analyze how the usage of rPosNet affects translation quality compared to multi-head attention. In Table 4, we depict the translation quality on EN→DE when combinations of encoder self-attention (enc-self), decoder self-attention (dec-self), and decoder cross-attention (dec-cross) employ multi-head attention (✗) or rPosNet (✓). The model using rPosNet only for cross-attention while all other layers employ multi-head attention (row 4) significantly decreases translation quality by 5.7% relative to the Transformer. The result suggests that content-dependent patterns incorporated by β cannot sufficiently capture source-target token interactions. We hypothesize that part of the reason is the inability of β to express varying relations across source tokens (see Section 3). While this may be a significant limitation of gating, we leave the exploration of this and other possible reasons to future work.

Table 4: A translation quality comparison of all combinations in which encoder self-attention (enc-self), decoder self-attention (dec-self), and/or decoder cross-attention (dec-cross) use either multi-head attention (✗) or rPosNet (✓). We conduct the experiments on EN→DE and report BLEU, BLEURT, and COMET.

rPosNet Layers			EN→DE		
enc-self	dec-self	dec-cross	BLEU	BLEURT	COMET
✗	✗	✗	26.3	71.1	47.6
✓	✗	✗	26.4	71.2	48.1
✗	✓	✗	26.1	71.1	47.2
✗	✗	✓	24.6	69.2	43.8
✓	✓	✗	**26.6**	**71.4**	**48.6**
✓	✗	✓	24.8	69.8	45.2
✗	✓	✓	24.3	69.0	42.8
✓	✓	✓	24.9	69.3	43.5

However, utilizing rPosNet within all self-attention layers (row 8), so that rPosNet is the only token-mixing method, does not lead to further degradation of translation quality with a relative degradation to the Transformer of 5.5% (5.3% relative in BLEU). Although the loss is substantial, rPosNet improves upon You et al. (2020)'s relative BLEU degradation of 12.3%[9]. Additionally, Table 4 shows that using rPosNet within decoder self-attention is only beneficial if encoder self-attention leverages rPosNet, whereas the usage within encoder self-attention always positively impacts translation quality.

7.3 From LightConv to rPosNet

With the similarities between LightConv and rPosNet, we want to understand what features of rPosNet are responsible for its better translation quality. While Wu et al. (2019) propose LightConv initially

[9]As reported by You et al. (2020)

Table 5: Starting from LightConv and progressively implementing the features of rPosNet.

Model	Params	EN→ZH		
		BLEU	BLEURT	COMET
Light Convolution	101M	33.2	63.4	40.6
+ GLU [LightConv]	104M	33.0	63.5	41.1
+ GeLU Gating	104M	33.5	63.7	42.4
+ Global Context	104M	33.6	63.9	42.4
rPosNet	104M	**33.8**	**64.2**	**43.1**

with the GLU mechanism (Dauphin et al., 2017) (see Equation 14), we differentiate between LightConv with and without GLU since the effect of gating is a central component of our analysis. We start with LightConv without GLU, denoted Light Convolution, and progressively implement the features of rPosNet. In Table 5, we show the translation quality on EN→ZH of the models leveraging the respective position-based approach instead of self-attention. Light Convolution (row 1) shows similar translation quality to LightConv (row 2). Replacing GLU gating with the gating mechanism of Section 2.3, denoted GeLU gating (row 3), increases translation quality noticeably by 0.5 points in BLEU, 0.2 points in BLEURT, and 1.3 points in COMET. Additionally, adding global context (row 4) by spreading the outer kernel weights across the whole sequence increases translation quality slightly by 0.1 BLEU and 0.2 BLEURT (no improvement in COMET). The remaining difference to rPosNet (row 5) is the different training scheme and rPosNet's unshared kernel weights across query positions. Together they add additional 0.2 points in BLEU, 0.3 in BLEURT, and 0.7 in COMET. The results show that all differences between LightConv and rPosNet are responsible for their translation quality difference. While the global context seems negligible for machine translation, GeLU gating, training scheme, and unshared token-mixing weights are the most important.

8 Related Work

The question of how to represent position and integrate it into the Transformer architecture has been a vast research field that we briefly want to overview and connect to our approach. An extensive line of research focuses on improving position embeddings (Kitaev et al., 2020; Liu et al., 2020; Kiyono et al., 2021) and their integration into the word vectors (Neishi and Yoshinaga, 2019; Wang et al., 2020). This direction is mainly orthogonal to our approach, and many ideas and methods can be leveraged with position-based attention. We leave these investigations for future work and restricted to learnable (Gehring et al., 2017) and sinusoidal (Vaswani et al., 2017) embeddings.

A different line of research focuses on integrating position within the attention mechanism (Shaw et al., 2018; Dai et al., 2019; Dufter et al., 2020; Huang et al., 2020; Raffel et al., 2020; Ke et al., 2020; He et al., 2021; Wu et al., 2021). They all improve over Transformer models for various tasks by modifying word and position interactions within the attention matrix and introducing relative position representations as a scalar or vector. While they still rely on content-dependent attention weights, they showed the importance of relative position representations, which we also used in rPosNet. However, we are interested in studying purely position-based self-attention approaches and how they can perform at least on par with the (content-based) Transformer. Additionally, we compare with Shaw et al. (2018) as an upper bound since it leverages token-token interactions and was proposed for machine translation.

9 Conclusion

We have introduced the gated token-mixing approaches aPosNet and rPosNet in order to find a high-quality self-attention alternative for machine translation whose attention weights can be pre-computed at inference time. Although their token-mixing weights are position-based, the gating mechanism introduces content dependency in the form of latent weights β, as shown by our analysis. These weights capture token-token interactions and are crucial for the results of rPosNet. In our experiments, we have compared aPosNet and rPosNet with existing position-based token-mixing approaches and found that rPosNet outperforms all the position-based alternatives and performs on par with (Shaw et al., 2018) on most benchmarks while saving more than 20% of the self-attention parameters. Moreover, the possibility of pre-computing rPosNet's token-mixing weights paves the way for high-quality machine translation on specialized hardware accelerators.

Limitations

The goal of this paper is to find alternatives for self-attention with minimal or no quality loss that

can pre-compute token-mixing weights at inference time. We have compared numerous approaches across many data conditions and model sizes to show the validity of our results. However, we can identify the following limitations in our work:

- rPosNet's position-based attention is an effective replacement of Transformer's self-attention, but its usage in cross-attention leads to quality loss;
- We did not have enough computational resources to run our numerous experiments multiple times, so we relied on the consistent results we obtained across different conditions and metrics.
- While our work is motivated by future use in memristor-based devices, we have no experiments in that specific hardware because i) it is still experimental and hard to find, and ii) our proposed models still contain operations that cannot be performed naively in the analog domain.

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A Formulas describing related position-based token-mixing approaches

In the following, we provide the formulas describing how the position-based token-mixing approaches from Section 2.2 formulate the context vector.

FNet

$$c_n := \mathcal{R}\Big(\sum_m \exp\big[-2\pi j \frac{n \cdot m}{M}\big] \cdot \mathcal{F}_h(x_m)\Big) \tag{11}$$

GaussianNet

$$c_n := \frac{1}{\sigma\sqrt{2\pi}} \sum_m \exp\Big[\frac{-(m-\mu(n))^2}{2\sigma^2}\Big] \cdot (W^V x_m) \tag{12}$$

LinearNet

$$c_n := \sum_m W_{nm} \cdot (W^V x_m) \tag{13}$$

LightConv

$$c_n := \sum_{k=0}^{2K} \frac{\exp W_k}{\sum_{k'=0}^{2K} \exp W_{k'}} \cdot \sigma_{\text{GLU}}(W^V x_{n+k-K}) \tag{14}$$

B Reformulating the gating mechanism with layer normalization

Substituting $z_m = \sigma_g(v_m)$ we rewrite the gating mechanism of Equation 4 as

$$\hat{c}_n := \Big[\sum_m \alpha_{nm} \cdot \text{Norm}(z_m)\Big] \odot g_n. \tag{15}$$

Similar to Section 3, we aim to rediscover the weighted sum over v_m. For this, we utilize the definition of layer normalization:

$$\text{Norm}(x) := a \odot [f_1(x)x - f_2(x)] + b, \tag{16}$$

Table 6: Comparing how different attention approaches leverage gating.

Model	Gating	Params	EN→DE		
			BLEU	BLEURT	COMET
Transformer	✗	66.5M	26.3	71.1	47.6
	✓	69.7M	26.6	71.6	49.6
Shaw et al. (2018)	✗	66.7M	26.3	71.4	48.6
	✓	69.9M	26.7	71.8	49.5
Rel. Self-Attention	✗	63.6M	25.7	70.4	46.4
	✓	66.7M	26.5	71.3	49.0
aPosNet	✗	61.8M	25.4	69.4	42.6
	✓	65.0M	25.9	70.6	46.1
rPosNet	✗	60.8M	25.3	70.1	45.1
	✓	63.9M	26.6	71.4	48.6

with gain $a \in \mathbb{R}^D$, bias $b \in \mathbb{R}^D$, $f_1(x) = \frac{1}{\sqrt{\sigma_x}}$ and $f_2(x) = \frac{\mu_x}{\sqrt{\sigma(x)}}$. The insertion into Equation 15 gives us:

$$\begin{aligned}\hat{c}_n \approx a \odot \sum_m \alpha_{nm} \underbrace{f_1(z_m) \cdot g_n \odot \sigma_s(v_m)}_{\beta_{nm} \in \mathbb{R}^D} \odot v_m \\ - a \odot \sum_m \alpha_{nm} \cdot f_2(z_m) \cdot g_n + \sum_m \alpha_{nm} b \odot g_n.\end{aligned} \tag{17}$$

Utilizing the normalization property $\sum_m \alpha_{nm} = 1$ we can simplify Equation 17 to:

$$\begin{aligned}\hat{c}_n \approx a \odot \sum_m \alpha_{nm}\beta_{nm} \odot v_m \\ + g_n \odot \Big[b - a \sum_m \alpha_{nm} f_2(z_m)\Big],\end{aligned} \tag{18}$$

with

$$\beta_{nm} = f_1(z_m) \cdot g_n \odot \sigma_s(1.702 v_m). \tag{19}$$

Although Equation 18 assumes α to be normalized, not normalizing α does not affect β and only adds a context-dependent scale in front of b. All in all, the Equations show that with or without layer normalization, gating introduces the token-mixing weights β.

C Theoretical complexity comparison

We compare theoretical complexities across position-based token-mixing approaches, the Transformer, and Shaw et al. (2018) concerning the number of operations and parameters in Table 7.

Table 7: We compare the theoretical complexity and number of parameters per attention layer. $\hat{K}$ refers to the bidirectional context size. With the formulation of position-based attention, the attention energies can be pre-computed after training, resulting in different complexities between training and search.

Model	Parameters		Operations	
	Train	Search	Train	Search
Transformer	$4D^2$		$2N^2D + 4ND^2$	
Shaw et al. (2018)	$\hat{K}D + 4D^2$		$\hat{K}ND + 2N^2D + 4ND^2$	
FNet	$2D^2$		$N\log(N)D + D\log(D)N$	
GaussianNet	$2D^2$		$\hat{K}ND + 2ND^2$	
LinearNet	$HN^2 + 2D^2$		$N^2D + 2ND^2$	
LightConv	$H\hat{K} + 3D^2$		$\hat{K}ND + 3ND^2$	
gLinearNet	$HN^2 + 3D^2$		$N^2D + 3ND^2$	
aPosNet	$5D^2$	$HN^2 + 3D^2$	$2N^2D + 5ND^2$	$N^2D + 3ND^2$
rPosNet	$\hat{K}D + 4D^2$	$H\hat{K}N + 3D^2$	$\hat{K}ND + N^2D + 4ND^2$	$N^2D + 3ND^2$

D Table: The impact of gating and query-key information

By depicting COMET scores in Figure 2, we visualized how the effectiveness of gating decreases with increased token-mixing weight expressiveness. In Table 6, we provide the full results with the number of parameters, BLEU, BLEURT, and COMET.

E Example failure cases of BLEU

Throughout our analysis, we observed that BLEU often disagrees with the semantic metrics BLEURT and COMET. For example, the translation quality in the Base configuration on EN→DE of GaussianNet, LinearNet, (see Table 2), aPosNet without gating, and rPosNet without gating (see Table 6) is similarly measured by BLEU but varies significantly in BLEURT and COMET. We analyzed translation samples of GaussianNet and LinearNet (see Table 8) and observed that BLEU often falsely depicts translation quality when hypotheses have little overlap with the reference or changing a single word alters the meaning of the sentence. While the inaccuracies of BLEU are already known (Kocmi et al., 2021), we want to show exemplarily how BLEU would have misled our analysis. Without using BLEURT and COMET, we would have concluded that aPosNet and rPosNet would be equally good without gating and that the hard-coded weights of GaussianNet are as good as the learnable weights of LinearNet.

Table 8: Example failure cases on EN→DE in which BLEU depicts a misleading score. These inaccurate BLEU scores are best visualized when comparing GaussianNet and LinearNet. Both models achieve the same corpus-level BLEU score but differ significantly in BLEURT and COMET (see Table 2). The translations show that measuring the syntactical overlap between the hypothesis and reference translation is not an accurate measure of translation quality.

		BLEU	BLEURT	COMET
Source	Haigerloch: Focus on the Abendmahlskirche			
Reference	Haigerloch: Abendmahlskirche rückt in den Blickpunkt			
LinearNet	Haigerloch: Fokus auf die Abendmahlskirche	15.2	84.0	72.3
GaussianNet	Haigerloch: Focus on the Abendmahlskirche	15.2	35.6	−15.0
Source	Does he know about phone hacking?			
Reference	Weiß er über das Telefon-Hacking Bescheid?			
LinearNet	Weiß er von Telefonhacking?	15.8	80.2	72.5
GaussianNet	Kennt er über Telefon-Hacking?	17.0	38.2	8.8
Source	The new season in the Falkenberg "Blue Velvet" club has begun.			
Reference	Die neue Saison in der Falkenberger Discothek "Blue Velvet" hat begonnen.			
LinearNet	Die neue Saison im Falkenberg "Blue Velvet" Club hat begonnen.	33.1	75.3	85.7
GaussianNet	Die neue Saison im Falkenberg "Blue Velvet" hat begonnen.	53.7	72.2	74.5
Source	Finally, let's talk pumpkins.			
Reference	Aber kommen wir endlich zu den Kürbissen.			
LinearNet	Abschließend möchte ich noch auf die Kürbisse eingehen.	4.8	71.4	41.0
GaussianNet	Schließlich, lassen Sie uns reden Kürbisse.	5.5	36.0	−60.3
Source	A combined English literature and language course will be scrapped.			
Reference	Der kombinierte Kurs aus englischer Literatur und Sprache wird abgeschafft.			
LinearNet	Eine kombinierte englische Literatur und Sprachkurs wird verschrottet.	9.6	60.8	44.0
GaussianNet	A combined German literature and language course will be scrapped.	3.7	19.8	−42.8
Source	However, there was no sigh of relief to be heard from Ludwigsburg.			
Reference	Ein erstes Aufatmen war aus Ludwigsburg dennoch nicht zu vernehmen.			
LinearNet	Von Ludwigsburg war jedoch kein Seufzer der Erleichterung zu hören.	5.3	76.7	46.7
GaussianNet	Es gab jedoch keinen Seufzer der Erleichterung, von Ludwigsburg gehört zu werden.	3.7	45.1	−30.3
Source	Sayings come from the Bible			
Reference	Sprichwörter kommen aus der Bibel			
LinearNet	Sprichwörter stammen aus der Bibel	42.7	90.3	108.0
GaussianNet	Sayings kommen aus der Bibel	66.9	60.7	3.7
Source	Uwe Link has an offer for anyone who wants to set off in a carriage.			
Reference	Wer dann mit der Kutsche vorfahren will, für den hat Uwe Link ein Angebot.			
LinearNet	Uwe Link hat ein Angebot für jeden, der in einer Kutsche starten will.	9.0	70.0	59.0
GaussianNet	Uwe Link hat ein Angebot für jeden, der einen Wagen starten möchte.	8.5	46.3	−10.0
Source	Solicitors should uphold the highest standards of integrity and should instil trust and confidence in the public.			
Reference	Anwälte müssen die höchsten Standards an Integrität aufrechterhalten und in der Öffentlichkeit für Vertrauen und Zuversicht sorgen.			
LinearNet	Die Staatsanwälte sollten die höchsten Standards der Integrität wahren und Vertrauen in die Öffentlichkeit schaffen.	10.9	77.9	67.4
GaussianNet	Die Umweltschützer sollten die höchsten Standards der Integrität einhalten und Vertrauen und Vertrauen in die Öffentlichkeit schaffen.	12.2	53.6	−0.7